Investigation of quadrature sets mapping algorithm in 1-dimensional S_N calculation.

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A new method to conduct the Discrete Ordinate (S_N) calculation with different orders of quadrature set is developed by dividing the angle space into sub-angular spaces. The neutron current conservation is considered. Investigation about this method is conducted with 1-D planar system. The preliminary result indicates this new method can give reasonable result on scalar neutron flux.

Keywords: S_N method, quadrature set mapping, neutron transport calculation, one-dimensional planar

1. Introduction

In reactor physics field, the Discrete Ordinate Method (S_N) is widely-used, and it is well-known that the prediction of the S_N relies on the spatial angular discretization as well as the order of quadrature set. Using different orders of quadrature set for different regions and the quadrature sets mapping is carried out on boundary, and then the computation burden can be reduced consequently. Although this idea was studied previously[1], follow-up research is lack. The authors develop an algorithm with a new idea of sub-angular space partial current conservation. Its performance is investigate based on a simple 1-Dimensional planar problem.

2. Method, numerical calculation and result

A relatively higher-order quadrature set should be applied in regions with pronounced anisotropic characteristics. One the other side, a relative lower-order quadrature set is acceptable for regions without significant anisotropy characteristic. The higher-order quadrature set can be marked as S_H and the lower one is marked as S_L.

The calculation algorithm can be summarized as follows. (a) The entire angular space $[0, \pi]$ is discretized into L parts symmetrically according to the lower-order quadrature set, each part is regarded as one sub-angular space. (b) The S_N calculation for both orders are carried out according to the sweep direction and the applied quadrature set of each mesh. (c) The quadrature set mapping is conducted when sweep calculation reaches to the boundary where different quadrature sets are employed. (d) Legendre polynomial interpolation is performed with ensuring the conservation of partial neutron current. The interpolation is performed for each half of the angular space, specifically $[0, \frac{\pi}{2}]$ and $[\frac{\pi}{2}, \pi]$. Within each sub-angular space, the following conservation relationship holds,

$$\eta = \frac{\sum w_i}{\sum w_j}, \qquad f = \frac{\sum w_i \tilde{\mu}_i \psi_i}{\sum \eta w_j \tilde{\mu}_j \psi_j}, \qquad \tilde{\psi}_j = f\psi_j,$$

where η is a normalization coefficient used for making the in-coming and out-going directions have the same weight, f is partial neutron current conservation factor, ψ_j is the results after the step of interpolation, $\tilde{\mu}$ is the cosine value of direction with respect to sub-angular space, and conservation factor f is used to give $\tilde{\psi}_j$.

A simple 100-mesh 1-D planar problem is designed where the left half employs a lower-order quadrature set and the right half employs a higher-order quadrature set. The left half is characterized by isotropic scattering, while the right half is characterized by anisotropic scattering, controlled by the HG-factor. The reference calculation is given by S_{32}. The following figures present the RMSE on scalar flux for each mesh and the relative difference in system-averaged scalar flux compared to the S_{32} calculation. The x-axis represents the order of S_L, and the legend labeled *previous* indicates results obtained using the same strategy as in the previous study[1].

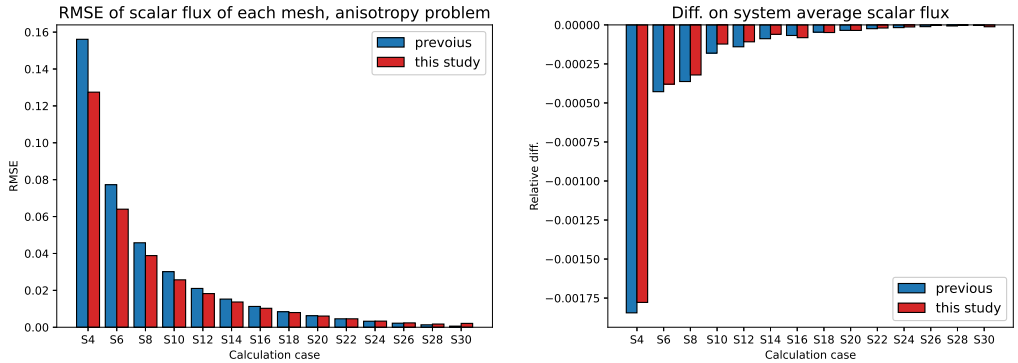

Figure 1: RMSE and average scalar neutron comparison for anisotropy problem.

3. Conclusion

The quadrature set mapping algorithm can predict scalar flux in a quite high accuracy in comparison with pure S_{32} even the S_L at a very low order. Besides, it is clear the sub-angular space partial current conservation strategy improves calculation accuracy in general, however, there are there are exceptions (for instance the $S_{16}-S_{32}$ case) indicating a more compressive investigation about ψ is necessary.

Reference

1. Jarrell, J. J., et al. (2009) Discrete ordinate mapping algorithm for region-based quadrature sets. In International Conference on Mathematics, Computational Methods and Reactor Physics 2009, M&C 2009 (pp. 2547-2555).