

A high-order weighted positive and flux conservative method for the Vlasov equation

*Takashi Minoshima¹, Yosuke Matsumoto²

1. Center for Mathematical Science and Advanced Technology, Japan Agency for Marine-Earth Science and Technology, 2. Chiba University

Kinetic plasma simulations are extensively utilized to investigate nonlinear physics in astrophysical, space, and laboratory plasma environments, including instabilities, wave-particle interactions, and particle acceleration. The Vlasov simulation is an Eulerian method that discretizes the distribution function to solve the Vlasov equation as an advection equation in phase space (Cheng and Knorr, 1976). While it can address numerical difficulties inherent in familiar Particle-In-Cell simulations and provide high-resolution solutions, its primary limitation is the substantial computational cost to allocate grid points in up to six-dimensional phase space, thereby necessitating high-order schemes for accurate solutions.

High-order Semi-Lagrangian methods for the advection equation are widely used in the Vlasov simulation (e.g., Sonnendrucker et al., 1999; Nakamura and Yabe, 1999). When employing high-order methods, nonlinear schemes are necessary to suppress numerical oscillations inherent in linear schemes. In Vlasov simulations, nonlinear schemes help to prevent the unphysical generation of plasma waves and negative densities in phase space. A drawback of using nonlinear schemes is the loss of accuracy, which can obscure physically meaningful profiles and degrade the conservation of energy and entropy.

In this paper, we introduce a new high-order semi-Lagrangian scheme that is conservative, positivity-preserving, and non-oscillatory, for solving the Vlasov equation. The scheme builds upon the third-order positive and flux conservative (PFC) method (Filbet et al., 2001), extending it to attain fifth-order accuracy. This is achieved by a convex combination of low-order polynomials in substencils, inspired by the weighted essentially non-oscillatory (WENO) scheme (Jiang and Shu, 1996). Unlike conventional WENO schemes, however, we utilize positive and non-oscillatory polynomials derived from the PFC method. This approach allows us to assign higher weights to substencils with larger L^2 norm, enhancing resolution. An approximate dissipation relation indicates that the spectral properties of the new scheme outperform those of the underlying linear scheme, improving both accuracy and stability. We apply this scheme to the Vlasov simulation to evaluate its performance, demonstrating improved conservation of energy and entropy.

Keywords: Vlasov equation, Advection equation, High-order scheme, Nonlinear scheme