

Accelerating Magnetohydrodynamic Models with Tensor Networks : Application of Density Matrix Renormalization Group

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Numerical simulations are essential tools for examining the impact of solar flares and coronal mass ejections on the Earth's magnetosphere. However, the inherent hierarchical structure of plasma and the coupling between different regions pose significant challenges for large-scale computations.

In recent years, tensor network methods have been actively studied in fields such as quantum physics and applied mathematics, which also face computational limitations. This approach decomposes a system into a network of smaller subsystems and evaluates the correlations between them. Unlike traditional methods, which treat fields and interactions on a fixed grid, tensor networks allow for a more global reconstruction of the system, enabling the extraction of essential information.

This methodology is rooted in the Density Matrix Renormalization Group (DMRG) (White, 1992), which was initially developed for quantum systems. In this approach, the subsystem of interest and its surrounding states are represented by a density matrix, whose eigenvalues are analyzed to extract significant information, which is then iteratively incorporated into subsequent subsystems. Since this technique is mathematically equivalent to singular value decomposition (SVD), recent developments in applied mathematics have led to algorithmic refinements and broader applications beyond quantum systems.

For example, in turbulence simulations in fluid dynamics (Gourianov et al., 2022), tensor networks have been employed to analyze vortex interactions in cascade processes. By evaluating the fields at each time step, this method successfully extracts the essential fluctuations that satisfy the Kolmogorov scaling law, allowing simulations to be performed using only the most relevant field components. As a result, memory usage is reduced to a logarithmic dependence on the number of grid points, and the overall computational cost is significantly lowered compared to conventional methods.

In this presentation, we will first introduce the research background and the tensor network approach adopted in this study. We will then discuss previous research, its application to magnetohydrodynamic models under incompressibility conditions, and the differences from conventional methods. Finally, we will explore the implications and future prospects of this approach.

Keywords: Tensor networks, Density-Matrix Renormalization Group (DMRG), Applied mathematics