

# Improving the Accuracy of the Collective Prediction by Maintaining the Diversity of Opinions: Preliminary Report

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This study aims to improve prediction accuracy by fostering diversity of opinions. We take an approach to give incentive to agents and induce diverse opinions and focus the minority reward system. The previous study assumes that the number of agents is sufficiently large, but the number of agents may be small in real-world situations. We show that the minority reward system is not necessarily efficient if the number of agents is small such as 100. To overcome this drawback, we propose a method to improve the performance by tuning the threshold for determining the minority and show the preliminary result of the evaluation.

## 1. Introduction

This study aims to improve prediction accuracy by fostering diversity of opinions. The idea of collective prediction, i.e., the idea of group formation of agents can be supported by the diversity prediction theorem that given a group (“crowd”) of predictive models, then the average squared error (collective error) is equal to the average individual error minus the variance between the individual signals (prediction diversity) [Page 07]. Forming a group whose diversity is large can make the collective error small. An approach for obtaining diverse opinions is to transform a prediction problem into a group formation problem [Lamberson 12].

Another approach is to give incentive to agents and induce diverse opinions [Mann 17]. Mann and Helbing propose an incentive scheme that rewards accurate minority predictions and show that this produces optimal diversity and collective predictive accuracy. They assume that the number of agents is sufficiently large, but the number of agents may be small. We show that the minority reward system is not necessarily efficient if the number of agents is small such as 100. To overcome this drawback, we propose a method to improve the performance by tuning the threshold for determining the minority and show the preliminary result of the evaluation.

## 2. Model

We consider a binary outcome,  $Y$ , which is the result of many independent factors,  $x_1, x_2, \dots, x_m$ . The outcome is determined by the sign of the weighted average of  $x_1, x_2, \dots, x_m$ .

$$Y = \text{sign}\left(\sum_{i=1}^m \beta_i x_i\right)$$

, where each contributing factor takes binary values such that  $x_i \in \{-1, 1\}$ . For example,  $Y$  is whether a stock price

goes up or down.  $x_1, \dots, x_5$  are an economic trend, interest rate, exchange rate, political situation, and corporate performance, respectively.

There are  $n$  agents. Each agent chooses one factor at a given time. An agent choosing factor  $x_i$  observes the value of  $x_i$  and then votes according to that observation. The collective prediction,  $\hat{Y}$ , is given by the sign of the collective vote  $V$ , which is calculated as follows.

$$\hat{Y} = \text{sign}\left(\sum_{i=1}^m \rho_i x_i\right)$$

Collective accuracy,  $C$ , is defined as the probability that the collective vote agrees with the ground truth given the distribution,  $\rho_i$ , of agents choosing each factor.

The reward function,  $f(z)$ , which determines the amount of the reward when an agent makes an accurate prediction as a function of the proportion,  $z$ . We consider the following three reward systems. In the minority reward system, agents are rewarded for an accurate prediction when the agents belong to the minority.

- Binary:  $f(z) = 1$
- Market:  $f(z) = 1/z$
- Minority:  $f(z) = 1 - \alpha H(z - \theta)$ , where  $H$  is the Heaviside step function.  $\alpha$  and  $\theta$  are control parameters.

Here,  $\theta$  determines whether the population is a minority or not. If the population selecting factor  $x_i$  is less than  $\theta$ , it is regarded as the minority. If  $\alpha$  is set to 1, only the minority get rewarded. If  $\alpha$  is less than 1, agents belonging to the majority also get paid. In the original minority reward system [Mann 17],  $\alpha = 1$ , and  $\theta = 1/2$ .

## 3. A criterion for determining the minority

In the study of Mann and Helbing [Mann 17], they assume that the number of agents is sufficiently large. If the number of agents is small, the followings may deteriorate the performance of the minority reward system.

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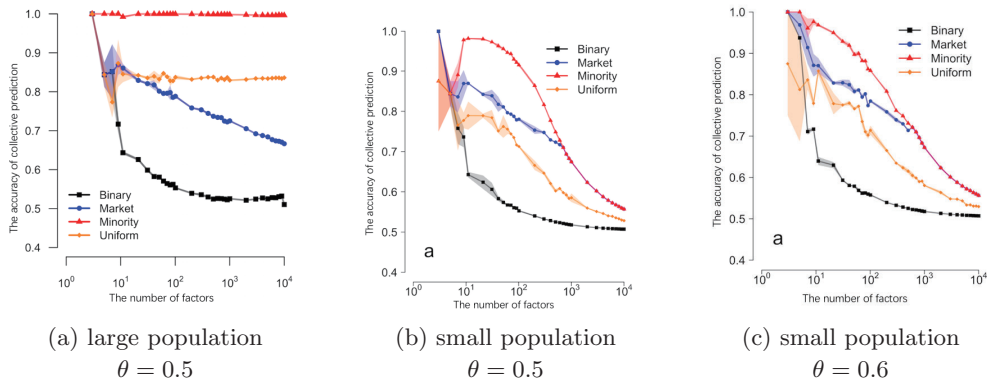


Figure 1: The accuracy of collective prediction

First, assume that the coefficients of only two contributing factors,  $x_1, x_2$ , are larger than zero. Here, the strategy of agents is represented by the probability of choosing  $x_1$ . An equilibrium is determined by the probability of choosing  $x_1$  and the performances of  $x_1$  and  $x_2$ . The performances of  $x_1$  and  $x_2$  are fixed. Thus, agents can change only the probability of choosing  $x_1$ . If the number of agents is large, fine-tuning can be achieved. However, if the number of agents is small, the tuning becomes coarse.

Second, when there are many factors whose coefficients are larger than zero, if the number of agents is small, the agents cannot cover all the factors. That is, only the factors having the high coefficients are chosen, and the factors having the low coefficients cannot be included in the prediction model, which deteriorates the performance of the collective prediction.

## 4. Experiments

To examine the observation in Section 3., we conducted the experiments. We compare the following three cases.

- Case1: Huge population, threshold  $\theta = 0.5$ .
- Case2: Small population (the number of agents  $n = 100$ ), threshold  $\theta = 0.5$ .
- Case3: Small population (the number of agents  $n = 100$ ), threshold  $\theta = 0.6$ . Also, pay 10% of the reward for agents belonging to the majority, i.e.,  $\alpha = 0.9$ .

In Figure 1, (a), (b), and (c) correspond to Case1, Case2, and Case3, respectively. In each graph, the horizontal axis represents the number of factors, and the vertical axis represents the accuracy of collective prediction. The results of the three reward systems mentioned above and *Uniform* are plotted. *Uniform* means a uniform allocation of agents to factors.

Figure 1 (a) is a reproduction of the results by Mann and Helbing [Mann 17] and shows that the minority reward systems always outperforms the other reward systems. Figure 1 (b), however, shows that the performance of the minority reward system drops and the market reward system is the best when the number of factors is less than ten. Also,

we can observe that the performances decrease when the number of factors increases, although the minority reward system outperforms the other reward systems.

In Figure 1 (c), the performance drop of the minority reward system when the number of factors is around 10 can be mitigated. The peak value of the accuracy in the minority reward system is around 0.97, and when the number of factors is equal to 1,000, the accuracy is 0.7.

This experiments confirmed our observation is correct and there is a possibility to improve the accuracy by tuning the threshold of determining the minority. Investigating how to find the optimal threshold is included in our future work.

## 5. Concluding remarks

We showed that the minority reward system is not necessarily efficient if the number of agents is small such as 100. To overcome this drawback, we proposed a method to improve the performance by tuning the criteria for determining the minority and showed the preliminary result of the evaluation.

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