Floquet Matrix Formulation for Dispersive and Dissipative Time-Varying Media

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1. Introduction

Photonic time crystals (PTC), consisting of periodic time-varying medium (TVM), have recently attracted considerable attention because it shows exotic properties that cannot be realized in static systems, such as magnetic-free non-reciprocity, Floquet harmonic generation, synthetic frequency dimensions, enhanced spontaneous emission rate and others. Hence it is critical to develop a unified framework to describe the dispersion relation and calculate the eigenmodes in PTC, which can be used to derive the topological invariants in TVM.

2. General Instructions

We consider the plasmonic materials with periodically modulated electron density, which can be realized via "shaking" the metal with pulse etc. The electron motion can be described by the equation of motion [1,2],

$$\left[\frac{\partial^2}{\partial t^2}+\Gamma\frac{\partial}{\partial t}+\omega_r^{\,2}(t)\right]\mathbf{P}(\mathbf{r},t)=\varepsilon_\infty\omega_p^2(t)\mathbf{E}(\mathbf{r},t)$$

where the plasma frequency and resonance frequency can be time-modulated, corresponding to the time-modulated carrier density and modulating parameter of meta atoms. By introducing the auxiliary field, we can formulate first-order Schrödinger like equation,

$$i\frac{\partial}{\partial t}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\\ \mathbf{P}\\ \mathbf{J}\end{pmatrix}=i\begin{bmatrix}0 & \varepsilon_\infty^{\;-1}\nabla\times & 0 & -\varepsilon_\infty^{\;-1}\\ -\mu^{-1}\nabla\times & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ \varepsilon_\infty\omega_p^2(t) & 0 & -\omega_r^2(t) & -\Gamma\end{bmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\\ \mathbf{P}\\ \mathbf{J}\end{pmatrix}$$

We then follow the Floquet theory, the wavefunction can be formulated as a linear combination of Floquet harmonics with frequency spaced by the modulation frequency. The spatial part is the solution of the static Hamiltonian. The wavefunction can be formulated in this form

$$|n,\alpha\rangle=\left(\tilde{\mathbf{E}}_{\alpha n}(\mathbf{r})\quad\tilde{\mathbf{H}}_{\alpha n}(\mathbf{r})\quad\tilde{\mathbf{P}}_{\alpha n}(\mathbf{r})\quad\tilde{\mathbf{J}}_{\alpha n}(\mathbf{r})\right)^T e^{in\Omega t},$$

$$\mathbf{\Psi}(\mathbf{r},t)=e^{-i\omega t}\mathbf{\Phi}(\mathbf{r},t)$$
$$\mathbf{\Phi}(\mathbf{r},t)=\sum_\alpha\sum_n c_{n\alpha}|n,\alpha\rangle$$

We first use the orthonormal condition for the basis, multiplying the left by $\langle\beta,m|$, The Floquet matrix can be formulated as

$$\hat{\mathbf{H}}_F=\begin{bmatrix}\hat{\mathbf{H}}_0+2\mathbf{I}\Omega & \hat{\mathbf{H}}_1(\mathbf{r}) & 0 & 0 & 0\\ \hat{\mathbf{H}}_{-1} & \hat{\mathbf{H}}_0+\mathbf{I}\Omega & \hat{\mathbf{H}}_1(\mathbf{r}) & 0 & 0\\ 0 & \hat{\mathbf{H}}_{-1} & \hat{\mathbf{H}}_0 & \hat{\mathbf{H}}_1(\mathbf{r}) & 0\\ 0 & 0 & \hat{\mathbf{H}}_{-1} & \hat{\mathbf{H}}_0-\mathbf{I}\Omega & \hat{\mathbf{H}}_1(\mathbf{r})\\ 0 & 0 & 0 & \hat{\mathbf{H}}_{-1} & \hat{\mathbf{H}}_0-2\mathbf{I}\Omega\end{bmatrix}$$

solving the eigenvalues and eigenvectors of the Floquet matrix, we get the quasienergy level and Floquet eigenstates of the PTC. With these eigenstates and biorthonormal relation, we can define the Aharonov-Andndan (AA) phase in PTC to locate the exceptional points (EPs)

$$\theta_\lambda=\int_0^{\frac{2\pi}{\Omega}}\left\langle\mathbf{\Psi}_\lambda^L(t)\,|\,i\frac{\partial}{\partial t}\,|\,\mathbf{\Psi}_\lambda^R(t)\right\rangle dt,$$

<u>*The results*</u>

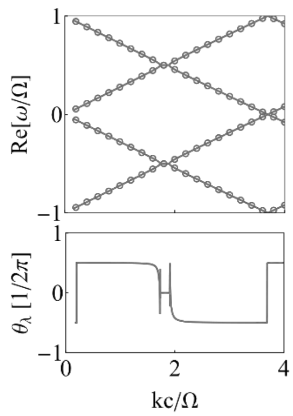

Figure 1. Quasienergy level of PTC with $\omega_p^2(t)=\omega_{p0}^2(1+\cos(\Omega t))$. The abrupt π phase change of AA phase indicating the location of EPs.

3. Conclusions

We have proposed a unified framework to calculate the quasienergy level and eigenstates in TVM. Using the eigenmodes, we define the Aharonov-Andndan phase and locate the exceptional points.

References

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