Different Weight-Related Interpolation Schemes for Isogeometric Analysis of Geometrically Nonlinear Beams
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Nghi Huu DUONG	Hokkaido University Thammasat University	nghihd.ce@gmail.com
Pruettha NANAKORN	Thammasat University	nanakorn@siit.tu.ac.th
Takashi MATSUMOTO	Hokkaido University	takashim@eng.hokudai.ac.jp

1. Introduction

Isogeometric analysis (IGA) [1] passed its early development period showing high efficiency and accuracy when compared to the traditional finite element analysis (FEA). IGA adopts the advantages of Computer-Aided Design (CAD) functions to approximate geometries and displacements. B-splines and their rational forms, specifically non-uniform rational B-splines (NURBS) [2], are among the most popular functions used in IGA. Nonetheless, when used for large displacement analysis, NURBS functions still have some shortcomings that can be improved. Concretely, the same NURBS basis functions are typically used to represent the initial configuration of a domain and all of its deformed ones. As these configurations can be very different, using the same basis functions to represent all of them is not appropriate. In the study of Vo et al. [3], this specific shortcoming is addressed for large displacement analysis of beams by treating the weights of control points as degrees of freedom (DOFs). Although the weights of control points are allowed to vary, the same set of weights is used for interpolating displacements and rotations.

This study builds upon the concept presented in the research work of Vo et al. [3] by treating the weights of control points as DOFs and utilizing distinct sets of weights for interpolating displacements and rotations. The study assesses the performance of this interpolation scheme by comparing its results with those obtained by the fixed-weight scheme and the scheme in the work of Vo et al. [3].

2. Interpolation schemes for Timoshenko beams

In this study, three different weight-related interpolation schemes are considered. Their details are given below. These schemes are applied to quadratic Timoshenko beam elements that use quadratic NURBS curves to represent their beam axes. The kinematics of the element, including its kinematic unknowns, are taken from the study of Chorn et al. [4]

Interpolation 1 is the conventional scheme in which only displacement and rotation degrees of freedom at control points are considered. The unknown vector for this interpolation scheme is expressed as

$$\mathbf{I}_1 = [u_{X1} \quad u_{Y1} \quad \varphi_1 \quad u_{X2} \quad u_{Y2} \quad \varphi_2 \quad u_{X3} \quad u_{Y3} \quad \varphi_3]$$

where φ_i is the rotation degree of freedom at the i^{th} control point, while u_{Xi} and u_{Yi} are the displacement degrees of freedom at the i^{th} control point.

The next scheme is Interpolation 2 where the weight of the second control point w_2 is treated as an additional degree of freedom. The unknown vector for this interpolation scenario is expressed as

$$\mathbf{I}_2 = [u_{X1} \quad u_{Y1} \quad \varphi_1 \quad u_{X2} \quad u_{Y2} \quad \varphi_2 \quad w_2 \quad u_{X3} \quad u_{Y3} \quad \varphi_3]$$

The third scheme is Interpolation 3 in which the weight of the displacement degrees of freedom at the second control point w_2 is distinguished from the weight of the rotation degree of freedom at the same control point $w_{2\varphi}$. These two weights, i.e., w_2 and $w_{2\varphi}$, are both treated as DOFs, leading to the following unknown vector:

$$\mathbf{I}_3 = [u_{X1} \quad u_{Y1} \quad \varphi_1 \quad u_{X2} \quad u_{Y2} \quad \varphi_2 \quad w_2 \quad w_{2\varphi} \quad u_{X3} \quad u_{Y3} \quad \varphi_3]$$

Fig 1 illustrates a beam axis represented by Interpolation 3. The performance of the elements using these three interpolation schemes is compared by solving a beam problem in the next section.

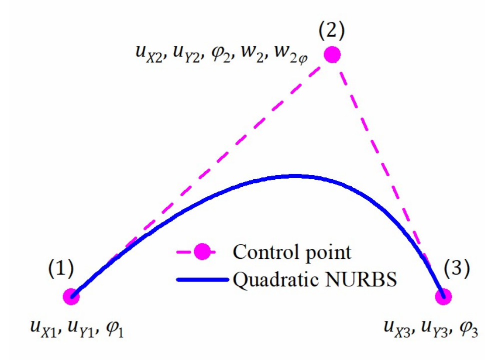

Fig 1. Interpolation 3

3. Numerical example

The problem considered in this study is an L-shaped beam under a uniformly distributed moment $m = 2\pi EI/L^2$ shown in Fig 2. Under the applied moment, the beam is deformed to the configuration illustrated in Fig 3.

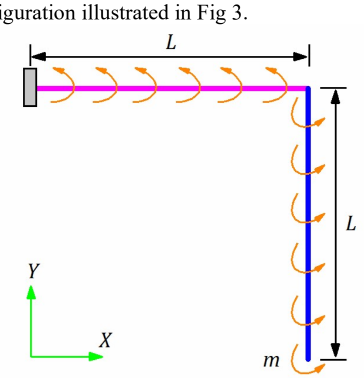

Fig 2. L-shaped beam subjected by a uniformly distributed moment

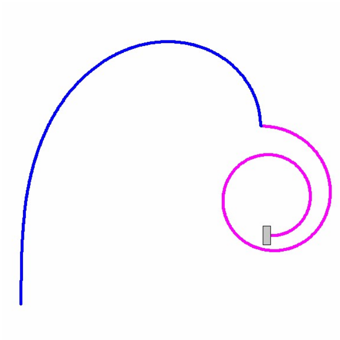

Fig 3. Final deformed configuration

The performance of the three interpolation schemes is compared in Fig 4 by using their relative errors of the deformations. The relative error is defined as

$$RelativeError = \sqrt{\frac{\int_0^l (\mathbf{r}_{num} - \mathbf{r}_{ext})^2 \, \mathrm{d}S}{\int_0^l (\mathbf{r}_{ext})^2 \, \mathrm{d}S}} \tag{1}$$

Here, S and l are the axial coordinate and length of the beam, respectively. In addition, $\mathbf{r}_{ext}$ is the exact deformed configuration and $\mathbf{r}_{num}$ is an obtained deformed configuration.

It can be seen from Fig 4 that treating weights as DOFs in the interpolations improves the obtained results quite significantly. As shown in Fig 4, without considering any weights as DOFs, Interpolation 1 does not yield as good results as the other two counterparts in terms of accuracy. Furthermore, by considering different weight unknowns for the displacements and rotation, Interpolation 3 yields better results than Interpolation 2 which considers the same weight unknown for the displacements and rotation. Even though the convergence rate of Interpolation 3 is lower than that of Interpolation 2 at the end of the DOF range, the difference is insignificant.

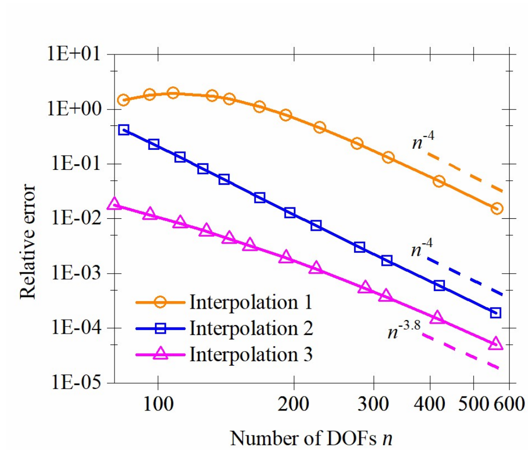

Fig 4. Relative errors of the deformations by the three interpolation schemes

4. Conclusions

Considering the weights of NURBS control points as DOFs can significantly improve the performance of the elements in terms of accuracy. In terms of the solution convergence, the convergence rates given by the three interpolation scenarios are similar at the end of the considered DOF range.

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