Symposium | A. Advances in Materials Theory for Multiscale Modeling

[SY-A9] Symposium A-9

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[SY-A9]Group-theoretical construction for constitutive equation of the first strain gradient elasticity

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Development of the theory of continuum mechanics is one of the central issues for quantitative understanding of multiscale phenomena. The first strain gradient elasticity (FSGE) is an attractive framework as it includes *non-locality* in the constitutive equation. Application of the theory for multiscale material modeling is however still limited due to the large number of independent strain gradient elastic constants. This study aims to overcome the difficulty by using the group theory. We conduct the irreducible decomposition for the 5th and 6th-rank strain gradient elastic constants, $\boldsymbol{M}$ and $\boldsymbol{A}$, under general linear group GL(3). The decomposition is based on the Schur-Weyl duality principle and Young symmetrizer technique in Frobenius algebra. For the isotropic and centro-symmetric case, the 6th-rank tensor is decomposed into three sub-tensors, $\boldsymbol{A} = \boldsymbol{A}^{\{6\}}+\boldsymbol{A}^{\{4,2\}}+\boldsymbol{A}^{\{3,2,1\}}$, whereas the 5th-rank tensor $\boldsymbol{M}$ vanishes identically. Here, the one-dimensional sub-tensor $\boldsymbol{A}^{\{6\}}$ is called the primitive symmetry class and satisfies the generalized Cauchy solid condition. The corresponding stress equilibrium equation yields a simplified 4th-order partial differential equation that is different from Aifantis’ Laplacian-type gradient theory. Another one-dimensional sub-tensor $\boldsymbol{A}^{\{3,2,1\}}$ denotes the elastic null-Lagrangian which is absent in the classical elasticity. The remaining three-dimensional sub-tensor $\boldsymbol{A}^{\{4,2\}}$ has two characteristic length scales and one of which has an imaginary value. We also revealed that the combined sub-tensors, $\boldsymbol{A}^{\{4,2\}}+\boldsymbol{A}^{\{3,2,1\}}$, with a proper kinematic constrain condition, reduces the well-known couple stress elasticity. Implementation of the Cauchy-type FSGE for NURBS-based isogeometric analysis (IGA) demonstrates that stress field around a point defect is regularized due to the non-locality included in $\boldsymbol{A}^{\{6\}}$.

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